Ekonomia — Wroclaw Economic Review 21/4 (2015)
Acta Universitatis Wratislaviensis
No 3713

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Marginal Unit vs. Marginal Unit — Some Additional Thoughts on the Differences between Menger, Jevons, and Walras

JEL Classification: B13, B31

Keywords: Menger, Jevons, Walras, marginalism, neoclassicism, Austrian school

Abstract

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Scholars usually trace the basics of the neoclassical theory back to the marginal revolution, in which three great thinkers amended the fundamentals of modern consumer and production theory. Scholars also recognize, however, important differences between those three thinkers' works — in the nature of the neoclassical framework and its application to the real world, especially in the field of political economy. In this note, we argue that the main difference in these works, not identified in previous publications on the subject of "dehomogenization", is their understanding of the marginal unit. We demonstrate the relevance of this important difference in the socialist-calculation debate.

In the introduction to *The Theory of Political Economy*, William Stanley Jevons stated explicitly that economics is and has to be a mathematical science and that his own theory is "purely mathematical in character" (Jevons 1965: 3). That economics is quantitative does not only imply the necessity of a mathematical treatment. According to him, quantities require a specific mathematical treatment, differential calculus:

> Nay, believing that the quantities with which we deal must be subject to continuous variation, I do not hesitate to use the appropriate branch of mathematical science, involving though it does the fearless consideration of infinitely small quantities. The theory consists in applying the differential calculus to the familiar notions of wealth, utility, value, demand, supply, capital, interest, labour, and all the other quantitative notions belonging to the daily operations of industry. As the complete theory of almost every other science involves the use of that calculus, so we cannot have a true theory of Economics without its aid. To me it seems that *our science must be mathematical, simply because it deals with quantities*. (Jevons 1965: 3, emphasis in the original)

As a consequence, Jevons built up his consumer theory using mostly derivatives (ibid.: 50–51)[1]. In the above quote we have a non sequitur that permeates modern microeconomics. It does not necessary follow from the fact that economics deals with quantities, that quantities must be treated mathematically, and surely it does not follow from the fact that their study requires a specific mathematical treatment: derivative calculus. Mathematics has different meanings. When a broad definition of mathematics is allowed, any kind of economic theory, even a linguistic one, could be considered mathematical, because the external world is based on regularities and orders of various sorts. Ordinal ranking of goods without arithmetical operations can express subjective utility in the mathematical form "A>B". In plain English this means A is worth more than B. There are no equations, derivatives, or probability distributions. Even the unit is gone. Yet this expression can also be seen as part of mathematics, since it demonstrates relations between certain objects (quantities) that can be expressed in symbols.

If economics deals with quantities and real existing objects and their relations to the social environment, then a science that grasps those relationships is in a broader sense mathematical. Jevons seems to be absolutely right in this respect. Even a simple process of buying can be described by the following equation: $d = f(p)$, where $f:P \rightarrow D$. Associated with the price of every good is a certain consumer demand. We can group the variables in pairs (price and quantity demanded) and describe this operation as a function[2]. These simple symbols, however, are analytical, and therefore express only what plain language already says. Strictly speaking, there are traces of mathematics even in literary economics, because every science assumes that the analyzed object may be mathematized to at least some extent. Following Jevons, then, since in economics we deal with quantities, we need some kind of mathematics to express the essential relationships.

Another example is the production function. There are two possible ways to present it: a simpler one, Cobb-Douglas, with homogeneous, costlessly interchangeable factors; and one more complex, though easier to understand, in literary language and describing the use of factors that are heterogeneous, that is neither perfectly substitutable nor perfectly complementary. The second, real-world type of production function presents an ordinal combination of factors, written down as in the notes of a journalist, without the use of complex math. The real-world production function could look like the following: a table is a function of specific types of wood, different workers' skills, a computer, paint, etc. This simple func tion represents a more complex world than the one in the Cobb-Douglas

[1] It comes as a surprise to see Stigler's statement (1937: 230) that Jevons's method was "essentially non-mathematical". It looks as if Stigler is following an assessment according to which a mathematical method is defined as using symbols and equations. The more equations one writes, the more mathematical his work is.

[2] Which would, of course, require more specifications than just a short equation.

function, yet both are mathematical in a broad sense, because they describe relations between quantities.

Accepting this definition with Jevons, we can confirm that economics is a mathematical science. But Jevonss did not stop there. Without justification he moved from the premise "we need mathematics" to the conclusion "we need differential calculus". We need proof that we must use a *particular kind* of mathematics. Léon Walras used an approach similar to Jevons's: he made the case for a marginalism dominated by equations. But not only does economics require mathematics. Economics itself is a branch of mathematics, says Walras (1984:70). We see this throughout *Elements of Pure Economics, or the Theory of Social Wealth*. (See for example his discussion of the theory of maximum utility and scarcity, ibid.: 119–127.)

Carl Menger, on the other hand, the last figure of the marginal revolution, was not interested in the use of equations, including those of differential calculus. He did not apply the notions of maximum and minimum values to explain the behavior of market prices[3]. Some authors even go as far as to state that his economics is of an institutional or disequilibrium nature (Streissler 1973: 172–173; Jaffé 1976: 520). A possible explanation for the lack of mathematics is Menger's ignorance; however, there is no strong evidence suggesting he was not familiar with it[4]. It is probable that his decision to avoid the use of algebra was for methodological reasons. Abstention from using formulas does not just mean using ordinary language. Contrary to Bloch (1940: 429), Menger and the others differed on the essence of a marginal unit, and consequently of marginal analysis[5].

In his path breaking contribution, Jaffé pointed out that in Menger, marginal utility is represented by integers. In Menger's framework there are also no first and second derivatives of utility (Menger 1950: 125–127; Jaffé 1976: 521)[6].

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[3] George Stigler was skeptical about Menger's approach, which he called a "weakness" that did not allow him to develop a "method by which the individual maximizes his want-satisfaction" (1937: 239).

[4] Jaffé (1935: 200) suggests that Menger was ignorant in this field. Mirowski also supports this thesis (1988: 22–23). Hayek, on the other hand, emphasizes that there is no reason to believe that (1934: 396–397). Howey states that Menger's library and his correspondence with Walras prove that he must have been familiar with mathematics (1973: 20–21). Menger's son, Karl, believes that his father understood the basic rules of mathematics, but he did not acquire an operative knowledge (Menger 1973: 45).

[5] Brems (1992) takes the same position as Bloch, and describes Menger's work using derivatives, and even goes as far as to say that "Austrians could not always see what they themselves were doing" (ibid.: 121). Krelle adheres to the same view (1973).

[6] "It seems clear Menger is thinking in terms of small, finite quantitative changes, and not of infinitesimals […]. The concept of small finite change is, of course, more realistic" (Stigler 1937: 241). Peart (1998: 308) argues convincingly that there are similarities between Jevons and Menger. This, however, does not apply to their method and the idea of marginal unit; an aspect that makes them poles apart.

A great champion of neoclassical orthodoxy, Paul Samuelson, considers Menger "the least important" of the triumvirate, and suggests he deserves to be neglected (1952: 61). The conflict between traditional neoclassicism and Menger's approach seems unavoidable. If we stayed faithful to the use of discrete marginal units, all "functions" in economics would become discontinuous; hence one could not use differential calculus. This would be a fatal blow to many models in modern microeconomics.

Mainstream microeconomics has chosen to go along with Jevons's unjustified use of the other type of mathematics. Instead of focusing on the marginal unit that is an object of action, it directs attention to infinitely small marginal units. These units appear only in the final state of rest, as described in the calculus equations.

The approach requires a completely different definition of the marginal unit and, consequently, a different notion of marginal analysis. As for Menger, Stigler was somewhat puzzled (1937: 241) to realize that using Menger's insights, the last apple possessed by the individual is more valuable than the next, unattained one (because actors strive to satisfy their most important ends before less important ones). In contrast, in neoclassical orthodoxy the marginal unit is irrelevant and small, such that it does not matter whether one adds it to their supply, or decreases supply by this amount[7].

The method for conceptualizing the marginal unit has important theoretical consequences. It leads, for instance, to a completely different theory of competition in Menger's work as compared to his contemporaries. Erich Streissler states that Menger (as a pioneer of imperfect competition) saw monopolies everywhere, and always emphasized the importance of product differentiation, which allows the producer to influence his price and not just face an infinitely elastic demand for his product (Streissler 1973: 169). The founder of the Austrian school undertook the task of explaining real competition in the world of uncertainty and heterogeneity of resources. This naturally led to the rejection of the mechanical approach present in Walras, Jevons, and later on in the perfect competition model of general equilibrium theory[8].

[7] Walras realized that there will be some problems when we deal with discrete units, not continuous utility curves. According to him, in such cases those will be "close approximations of the arithmetical mean of the intensities of the last wants satisfied and of the first wants unsatisfied" (Walras 1984: 144). Since there is no continuity of units of goods, and the marginal unit is discrete, a submarginal unit does not have the same value as a marginal unit. Hence Warlas's proposition is to count the arithmetical mean of submarginal value and marginal value. In Menger's framework these are and have to be different. We might also wonder whether Walras's theory of utility was ordinal as we teach nowadays in modern economics — what in fact could "arithmetical mean" in the above quote mean if utility is not subject to cardinal measurement?

[8] Streissler goes even further to suggest that Boehm-Bawerk and Wieser accepted perfect competition theory and deviated from his teacher in this aspect (1973: 170). Some parts of Boehm-Bawerk's writings might partially support this thesis (see for example his discussion of "fractional

One important implication of this difference is that it guides us indirectly to the socialist-calculation debate, which took place half a century after the marginal revolution. An important figure in that debate was Ludwig von Mises, a follower of Menger. Instead of being a small piece in an equation, the marginal unit is relevant to an entrepreneur or a consumer in seeking to arrive at a certain end. It could not be infinitely small, since that would imply that they may neglect it. When one pays attention to marginal amounts from this perspective, one can see how the institutional framework influences their productive employment.

Starting from Menger's insight, Mises developed a pricing theory based noton continuous functions, but on the actions of private property owners who appraise marginal units of capital goods according to their *perceived* best use (*perceived,* therefore not recognized objectively in pre-existing production functions).In Walras and Jevons, owners do not need to appraise the marginal unit, since it is infinitely small[9], as unimportant to actors as the last particle of water that goes into a drinker's glass[10]. If they were right, there should not be any problems with the pricing of productive goods, whether in capitalism or socialism, and Mises's argument could be rejected. If, on the other hand, the marginal unit is relevant, then it has to be appraised[11]. A legal framework for assessing marginal units might in fact be very important, and hence economic forces could work differently under capitalism and socialism. Institutions would matter.

Following Menger's account, Mises developed a theory of socialism. If the marginal unit is important, then it might also be important what it means to possess it and how it can be exchanged[12]. If it is infinitely small, then it is irrelevant (it does not even matter whether an actor adds it to or subtracts it from their sup-

wants into which out sensations of want are divisible" in Boehm-Bawerk 1959, vol. II, p. 139; thenhe goes back to discrete units).

[9] One additional point should be made. Even if we accept the neoclassical theory of a marginal unit, then formulas presented by Walras and Jevons are less general than Menger's. The statement expressed in plain language (greater quantity is valued more than smaller quantity) includes also the extremely unrealistic neoclassical equations based on continuous functions and perfectly divisible goods (see on this Menger 1973: 40). Hence, we should rightly conclude that contrary to modern views Menger was in fact saying more than his contemporaries. Mark Blaug states that discontinuity presents only a "formal, not a substantive, difficulty in the analysis" and rules are basically the same (1986: 297). Our analysis takes an exactly opposite approach. Walras (1984: 72) is lost in saying that his differential calculus expresses the same things as an ordinary language, but "more succinctly, precisely and clearly". Stigler (1946: 181) also doubly states that using continuous functions is "unrealistic only to a minor degree, and essential to analytic treatment".

[10] Although a particle is still discrete and even in here differential calculus is only an approximation.

[11] Let us briefly note here that irrelevance does not imply infinitely small units, but infinitly small units imply irrelevance.

[12] For Menger, possession of a good is a basic element in the general theory of a good (1950: 52), and later on plays an important role in the pricing process (the possession is connected to the property structure). Walras, on the other hand, from the very beginning wants to detach economics from these problems (1984: 68).

ply), since it already possesses an assigned role based on the differential equation. It is only being a part of such an equation that determines its allocation. One can thus understand Mises's argument against socialism, and his attempt at a realistic explanation of the pricing process under central planning. We can also see why the neoclassicals responding to Mises could not see the nature of the problem he posed. We can see this clearly in the case of one of his most important opponents, Oskar Lange, who in writing on the history of economic thought summarized the Austrian school's approach in the following way:

> [man] *maximizes* some amount, usually called utility. The *maximization* of utility is done with the help of marginal calculus, which is expressed here as a marginal utility *calculus*… Some of the leaders of the subjectivist movement, like Jevons and Walras, realized that. The representatives of the so-called "Austrian school"…did not know higher math. Introducing and applying marginal *calculus*, they somehow discovered again *differential calculus* (two hundred years after Newton and Leibnitz)… Marginal *calculus* is used did not only by the subjectivist school. This *calculus* is used anywhere, where we talk about *maximizing and minimizing* some amount with the use of differential calculus. (1980: 204–205, emphasis added, translation mine)[13]

Let us go back for a second to the production-function approach. Mises's arguments against socialism, or perhaps for the necessity of economic calculation, rest on the notion of heterogeneity of resources and the complexity of the world. Different types of capital goods could not be summed up in one unit of capital that the planners might use. Instead of aggregating the factors, financial and monetary calculation is necessary to appraise them and assign to them a monetary value, leading to effective allocation.

The problem is, however, that in assuming continuous Walrasian functions one also assumes the possibility of getting rid of heterogeneity of resources that lies at the heart of the calculation problem. Derivatives allow the planner to list the tiniest possible effects of marginal changes throughout the sectors of the economy. Now, those known changes of infinitely small units can be listed, ranked, and compared vis-à-vis each other. The comparisons can well be used to form shadow prices in Becker's sense (substitutes for market prices), signals that describe the influence of every marginal unit, of where it could be most efficiently employed, substituted for or assisted by a complementary factor.

Lange was so enthusiastic about this particular type of mathematics that he did not realize from Boehm-Bawerk's writings that prices are viewed by the alternate tradition as results of actions whose aim is to deal with concrete units of goods[14]. There is no derivative in the pricing process, but instead there are, to

[13] Schumpeter thought the same (1954: 18, 956). Consequently, he thought that socialism could easily work (1961: 172 ff).

[14] Although Boehm-Bawerk referred to goods, which can be divided, he usually referred to discrete units. His famous examples are horses, and obviously the idea of one tenth of a horse is absurd. Moreover, when one uses discrete units in the analysis, some natural problems arise. The example is Boehm-Bawerk's "spurious marginal utility" (1959, vol. II: 158). Howey (1960: 157) commented that those problems "would have disappeared if he had accepted the idea of complete divisibility".

use a Boehm-Bawerkian term, "marginal pairs", economic goods controlled by different individuals (Boehm-Bawerk 1959, vol. II: 220–221; see also Menger 1950: 204)[15]. Realizing this, we may infer that if Lange could not see the difference between Menger and others, then it would be even more difficult for him to adhere to Mises's argument about the need for a monetary assessment of marginal units under a private property regime[16].

As we see from the above note, the differences between Menger and other marginalists were not only significant during their lifetimes, but their importance extended through the century. Those were differences not only in formal tools, but also in their content. The Austrian marginal economics is not the neoclassical marginal economics not only because of the differences between their implications and frameworks, but especially because of the difference in assumptions and the idea of marginal unit. One consequence of that was seen in the case of the socialist-calculation debate, but a similar analysis could be applied to competition theory[17], monetary-demand theory, and other important economic issues. Either the marginal unit is to be seen as an important part of the pricing and imputation process, or it is to be seen as a mostly irrelevant unit determined by the previously known equation.

[15] Boehm-Bawerk was not critical of the differential expressions of economic theorems: "many economists are accustomed to employ in their presentations such mathematical symbols as will permit them to portray even composite and complex situations by means of simple forms and formulas. […] Thus, when buyers and sellers make continuously changing valuations — upward or downward, as the case may be — and these valuations represent offers to buy or sell partial quantities of a market good, there is a special predilection for depicting them by means of continuously ascending or descending curves […]. Now that is a perfectly unobjectionable procedure. And yet I still find it questionable whether […] this method of presentation is really capable of completely supplementing and making superfluous a description by running commentary of determination or price, such as I have undertaken to present" (1959, vol. II: 233). Moreover, note 23 (p. 436) shows how Boehm-Bawerk later on got convinced by Edgeworth that sometimes one could use perfectly divisible units in the analysis. Yet, if that is the case, then why should one not use differential equations in the theory of prices? Apparently, part of the reason for Boehm-Bawerk to avoid the calculus was pedagogical; having noticed the formulas "the reader stops understanding and is no longer interested in the book" (quoted in Kauder 1992: 99). A report written by Boehm-Bawerk (1959, vol. II.) includes a differential formula. Provided that Boehm-Bawerk understood the equation he had written, then he must have been familiar with mathematics. Stigler's view is that he was not trained in mathematics. He does not, however, provide an argument for this (1946: 181).

[16] One of the critics of a marginal theory lamented that a "hocus-pocus of marginal utility does not explain price" (Tugwell 1922: 330). More precisely it should be noted and addressed to equilibrium theorists that a "hocus-pocus of differential calculus does not explain price".

[17] For example: Baumol (1977: 511–512) notes that if all producers can influence the price of their goods (that is if there are relevant marginal sellers, who deal with discrete amounts of goods), then all welfare considerations of the neoclassical monopoly theory collapse. Hence, provided that an economy is discontinuous in its heterogeneous structure (it seems reasonable to assume so), the neoclassical idea of competition becomes very problematic. And so are its welfare considerations and theoretical basis for an antitrust policy.

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