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Formal logic and natural ways of reasoning

Abstract: In the paper I ask the question about the relation between formal logic and the natural logic of human mind. By a natural logic I mean the ways of thinking of a person that is intelligent but untrained in formal logic. As it turns out that the laws, rules or properties of formal logic in some cases diverge from the natural ways of reasoning, I explain the causes of this divergence. Since the majority of research in this area has been carried out from the standpoint of psychology, as a logician I suggest a slight change of the angle from which we look at the problem. I argue that certain narrowing of an interdisciplinary research would be helpful in getting a better picture of natural logic, and might provide a new stimulus for formal investigations.

Keywords: formal logic, informal logic, philosophy of mind, cognitive science

1. Logicians are often interested in the interdisciplinary researches concerning as many areas as computer science, artificial intelligence, linguistics or cognitive science. But they are very seldom interested in the psychology of reasoning, although researches of this kind have been carried out for decades (e.g. in the works by P.N. Johnson-Laird, such as *How We Reason*[1]). Perhaps the reason is that they do not expect much from this kind of results for their own work. However, psychological aspects of reasoning may be not only an invaluable inspiration but also a source of tangible hints that have been unnoticed or simply unknown to us so far. This is why I am interested in something that I call a 'natural logic'. By a natural

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[1] P.N. Johnson-Laird, *How We Reason*, New York 2006.

logic I mean the variety of ways of thinking used in an intellectual activities by a person that is exceptionally brainy but untrained in formal logic.

Although I ask the question about the logic we know intuitively and use in everyday life, it has to be stressed that it is not my intention to glorify it. Being aware of its imperfections, I look for its virtues in order to get the most out of them for my science, i.e. formal logic. What I am interested in is how the natural logic of human minds relates to formal logic and especially how much it differs from it; we know many of those differences, but obviously not all of them.

Some of the differences are discussed by Johnson-Laird, when he presents deductive reasoning in logic, artificial intelligence, and cognitive science.[2] As he points out, the difference is already at the level of language, since formal logic deals with sentences, while everyday reasoning concerns propositions. Ambiguities arise when a sentence expresses many different propositions. The theory of deductive reasoning is, according to him, incomplete, because it does not say which conclusions are sensible, and sensibility of conclusions is important in everyday reasoning.

Johnson-Laird mentions several phenomena, established experimentally, showing how logically untrained individuals reason in some situations. The evidence shows, for example, that deductive reasoning from exclusive disjunctions (A or B, but not both) is easier for a formally untrained person than reasoning from inclusive disjunctions (A or B, or both), which are used in formal rule theories. The use of counterexamples by which individuals spontaneously refute invalid inferences is another phenomenon mentioned by Johnson-Laird in order to show the difference between intuitive thinking and the formal rule theories which in general make no use of counterexamples.[3] There are also so-called 'illusory inferences'; they occur when people go wrong because they cannot cope with what is false (if the sentence *You can have the soup or else the salad, but not both* is false, then it is possible for you to have both the soup and the salad). Another aspect of natural logic is the use of knowledge in the process of reasoning. According to C.K. Riesbeck and R.C. Schenk or J. Kolodner, a particular inference is based on the memories of previous inferences.[4]

The subject of the psychology of reasoning, in its development, has mainly cognitive motivations. The psychological study of how people reason leads to numerous questions concerning rationality, intelligence or relationships between emotion and reason. One of the most interesting and conclusive works in this area is probably the book *Human Reasoning* by J.St.B.T. Evans, S. Newstead, and R.M.J. Byrne.[5] The authors examine conditional and disjunctive reasoning, rela-

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[2] P.N. Johnson-Laird, 'Deductive Reasoning,' *WIRE's Cognitive Science* 1 (2010), p. 8–17.

[3] There are, however, refutation rule systems, e.g. those developed by Tom Skura, see: T. Skura, 'Refutation Systems in Propositional Logic,' [in:] *Handbook of Philosophical Logic*, vol. 16, D.M. Gabbay, F. Guenthner (eds.), Dordrecht 2011, p. 115–159.

[4] C.K. Riesbeck, R.C. Schenk, *Inside Case-based Reasoning*, Hillsdale, NJ 1989. J. Kolodner, *Case-based Reasoning*, San Mateo, CA 1993.

[5] J.St.B.T. Evans, S. Newstead, R.M.J. Byrne, *Human Reasoning: The Psychology of Deduction*, Hove 1993.

tional inferences as well as reasoning with syllogisms and quantifiers. What they are interested in are the extent and limitations of human competence in deductive reasoning, the question why people make errors and are biased while reasoning, and the role of the context in which deductive problems are formulated.

As a logician I am interested in the rules of inference, the laws of logic, and the general properties of logical systems. Since the phenomenon of reasoning takes place in human minds, the question how natural ways of reasoning relate to the rules and laws seems to be a natural one. It goes without saying that what I search for are more precise and accurate formal solutions. For this reason I have to approach the problem from a different angle than it has been done before. I would like to know, for instance, when the limitations of human competence are not important and in which cases the errors are harmless. Getting to know the answers to these questions could perhaps help us modify the existing formal solutions in the way they have not been modified before. This approach has a different goal we strive for.

The examples I present below are well-known to logicians, although not all of those who work in the area of formal logic are aware of how much informal ways of reasoning differ from the formal ones. The full range of the difference is still unknown and for this simple reason it is worthwhile to take a closer look at the matter. Since the answers I am looking for cannot be found by logicians alone, the research psychologists' assistance, their science toolbox especially, may be crucial in finding out about our deductive reflexes. The main objective of this paper, then, is to issue an invitation both to logicians and psychologists to explore the matter again in a systematic way, but this time — hopefully — for the benefit of formal logic.

In the next section I give a short survey of several laws, rules and properties of formal systems in order to show the range of aspects of the natural-versus-formal issues one may expect. Among them the reader will find: the *modus ponens* rule, the conditional introduction law, explosiveness, the law of excluded middle, the principle of non-contradiction, *reductio ad absurdum*, the &-introduction rule, and monotonicity as the property of classical consequence operation. I also give two examples of the observations I made concerning students' intuitions about some logical issues: their approach to propositions that contain empty terms and to the problem of validity of arguments.

2. Has logic been discovered or invented? To answer this question is to decide whether logic has always been there and logicians have been recognizing it over the ages and describing its laws, rules and properties, or they have constructed it from scratch in order to provide a convenient deductive tool to the public. It seems that the answer to this question is that logic is being invented (and reinvented) all the time, since we have not discovered it entirely as yet. The logician's work is, in a way, very similar to that of the physicist: they both seek for a description of some basic natural phenomena in the form of laws and rules. Obviously, they make discoveries, but in order to get a fairly complete picture of the situation, they have to fill in the gaps with their inventions. An interesting question: "which part of

formal logic is a discovery and which is an invention?" cannot be answered without touching upon the subject of human thinking processes.

As it is known, N. Chomsky in his *Syntactic Structures* and later works developed a theory of innate linguistic abilities.[6] It is based on the observation that a child grasps the rules of a language with the speed and skill that cannot be explained by a learning theory. The fact that a child is able to formulate any number of new but correct sentences suggests that it has an innate linguistic disposition of the mind and a built-in structure of a universal grammar. The ability to think logically is most likely innate too. There are a lot of extremely logical people who have never taken a course in logic, as well as quite a few educated people (according to my observations) who have problems with logical thinking. The questions are: what kind of an innate logical structure rests in human minds and how it relates to the science of logic? I am perfectly aware that the questions I am asking go beyond logic, for they are also questions about the nature of the mind's activity, and therefore it is difficult to answer them without not only a psychological but also a neuroscientific insight. At this stage I do not aspire to answer them fully but some observations that can be easily made show that there are substantial differences between those two entities — the natural and formal logic (see also A. Kisielewicz *Logika i argumentacja*, where a similar point is made[7]).

Natural logic of human mind has its numerous formal approximations. Relevant, temporal, paraconsistent, non-monotonic, and many other logics more or less successfully formalize the nuances of our mind-work. But it does not seem that there is one and only non-classical logic according to which human minds work. Instead, it seems that, in different situations, people reason in ways that can be formalized by different non-classical logics. Each of them has its own precisely specified motivation. Non-classical logics are not the only formalizations that are close to natural logic. Statistical reasoning, the reasoning of probabilities or possibilities issue alternative ways of approaching the problem, and have brought many illuminative results.[8] But in these cases, the validity criterion for reasoning has to be replaced; we consider probabilistic conclusions instead of valid deductions. But are we not sacrificing too much in this approach? The validity criterion seems a natural one, we often think: if it is true that A, then B should be the case (should be true).

Logic in its basic meaning is the formal theory of reasoning. Logicians are interested in searching for the sets of rules of inference that make it possible for formal reasoning to be expressed and analysed. The rules are supposed to be truth-preserving, i.e. to satisfy the validity criterion. This means that if we start our reasoning with truths, and use arguments of the form of our rules only, then we shall get only truths at the end of the reasoning. One of the most obvious rules of inference is *modus ponens* (the rule of detachment), which can be expressed by

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[6] N. Chomsky, *Syntactic Structures*, The Hague 1957.

[7] A. Kisielewicz, *Logika i argumentacja, praktyczny kurs krytycznego myślenia*, Warszawa 2017.

[8] M. Oaksford, N. Chater, *Bayesian Rationality: The Probabilistic Approach to Human Reasoning*, New York 2007.

the following diagram (the letters A and B represent sentences, and the arrow represents "if ..., then ..."):

A → B
A

B

It is truth-preserving because if we assume that the premises A → B and A are true, then, according to the definition of the conditional, the conclusion B has to be true as well. If John was in Paris, John saw the Eiffel Tower. John was in Paris. Therefore, John saw the Eiffel Tower. The *modus ponens* rule is a good example to begin with; it is hard to find a logician or a non-logician who would question its reasonableness or naturalness. It seems that not only is it formally valid and derivationally useful but also it is well-founded in our minds. Even a small child knows very well what to do in order to get an ice-cream when told by her mother: If you behave correctly, I'll buy you an ice-cream.

The law of conditional introduction, that is the formula A → (B → A), is an example of the basic logical law that is somewhat odd as far as our usual way of thinking is concerned. It is beyond doubt one of the most important laws of logic, the reason being that it is hard to think of a convenient axiom system for classical or intuitionistic logic without it. In order to better understand the way it works, it is convenient to derive on its basis the following rule of inference.

A

B → A

It is obviously truth-preserving, which means that it is a formally acceptable and valid argument form. It says that if we accept the sentence A (as true or justified), then we can precede it with any sentence B, and the conditional obtained this way has to be accepted too. But in this case, when we start with a true sentence, we may obtain a totally unexpected statement that has to be recognized as true as well. To demonstrate this, let us take a true sentence for the letter A, and any sentence for the letter B, for example:

A — People die, when they are old.
B — People read books.

Starting with a true premise People die, when they are old, we come to the conclusion that If people read books, then people die, when they are old, which we have to accept as a true sentence, since the argument is truth-preserving. But common sense suggests disagreement with the fact that reading books kills. Moreover, since we have a free choice of B, we can take a sentence that has nothing to do with A (e.g.: If 2+2 = 5, then people die, when they are old). We also have to recognise as true even paradoxical statements such as:

If people are immortal, then people die, when they are old. It turns out that in some cases we do not have to assume anything about the antecedent of a conditional in order to make it sufficient to entail the consequent. This particular pattern of reasoning is regarded by some logicians as one of the so-called paradoxes of implication and together with others was the reason for introducing logics that require that the antecedent should be "relevant" to the consequent and the premises should be "relevant" to the conclusion. The relevance requirement is explained in various ways: for example, the premises should be inconsistent with the negation of the conclusion or they should have something in common with the conclusion (see A.R. Anderson. N.D. Belnap Jr, *Entailment, The Logic of Relevance and Necessity*[9]).

There are more laws and rules of logic that do not obey the relevance requirement. One of them is the explosiveness, that is the rule: A and ¬A, therefore B. The way it works is that if we accept two contradictory statements, then we have to accept any possible statement, including an absurd one. But the contradictory statements A and ¬A may have nothing to do with the statement B. This feature of classical logic makes it impossible to apply it to inconsistent sets of information. Once we accept a contradiction, the theory considered becomes trivial, since any statement we can express in the language turns out to be its theorem. In order to avoid triviality and still remain logical, we need a paraconsistent, that is a non-explosive logic. Is it a very special, unusual logic? Not at all. Common sense reasoning usually prevents us from deriving an infinite string of conclusions from two contradictory pieces of information. The non-explosive logic we are using in situations like that turns out to be a convenient tool which simply prevents us from being driven to despair.

The law of excluded middle asserts that whatever A may be, everything must be A or not-A (symbolically: A ∨ ¬A). According to this law each natural number is even or not-even, Napoleon was German or Napoleon was not-German, etc. It is hard to question its validity. But, as J. Jeans notices (finding an interesting interpretation of the sorites paradox), this law entails some strange consequences when properties of things that are considered have a gradual character.[10] Since a man is either young or not-young at every moment of his life, we conclude that the transition phase from young to not-young must occur in an instant (and so does the transition from an uneducated to an educated person, and from a not-ripe to a ripe fruit). We arrive at a paradox here because in real life properties of things or human beings are somewhere between A and not-A, that is, in the area which is excluded by virtue of the law. This way a law of logic that plays an extremely important role in formal deductive systems, loses its significance in every day instances of reasoning. Another example of this kind is the principle of non-contradiction: ¬(A&¬A). It states that no object can have and not have the same feature at the same time (or two contradictory statements cannot be true at

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[9] A.R. Anderson, N.D. Belnap Jr., *Entailment, The Logic of Relevance and Necessity*, vol. 1, Princeton, NJ 1975.

[10] J. Jeans, *Physics & Philosophy*, Cambridge 1942, p. 93–95.

the same time). The principle raises no doubts when considered from the mathematical or philosophical viewpoint (unless one has dialectical inclinations) but it loses its importance in real life situations, when one has to face vague, fuzzy or uncertain information.

Considering the basic laws and rules of logic, one cannot disregard the *reductio ad absurdum* rule (RAA). The RAA rule rests on the natural principle of reasoning that if we can derive a contradiction from a proposition A, then A has to be refuted, therefore we are entitled to conclude its negation. In a more formal way it can be expressed as:

Given X, A ⊢ B&¬B, we may have X ⊢ ¬A

where the symbol ⊢ is read as 'therefore' (before it, the assumptions are listed, and after it we write the conclusion).

This is probably the most powerful and one of the most useful of all logical tools in the area of mathematics. Mathematicians use it and abuse it without even referring to its wording, let alone justifying its validity. They simply take it for granted. One of the first theorems that a student of mathematics has an opportunity to get to know is the fact that √2 is not a rational number. To prove it, a lecturer assumes that √2 is a rational number and after a few steps of deduction he or she arrives at a contradiction; the conclusion is that √2 is an irrational number. But a student is never told why we are allowed to proceed this way. Non-mathematicians use the RAA method unconsciously in their every day reasoning, however, in order to assert their disagreement with the assumption, they aim to derive an absurd statement rather than a one-and-zero contradiction as mathematicians do. Informally used, *reductio* works this way: if we can show that somebody's opinion leads to an absurd one, then nobody will take that initial opinion seriously (e.g. during a national dispute that we had several years ago in Poland the idea that unemployed women should be paid by the government for their housework, eventually, in the course of the discussion, came down to the proposal that men should be paid for their DIY activities and schoolchildren for doing their homework).

It is not an easy task to justify RAA as it is formulated here. First of all, it is not a rule of inference, but a meta-rule of inference. Since its premise and conclusion are not formulas but sequent-expressions, the logical values 'true' or 'false' cannot be assigned to them, so that we cannot even consider *reductio* as truth-preserving (in its application, we pass from one sequent to the other). In order to show its validity, we have to move on to the meta-level of considerations, that is to show that if the premise is truth-preserving, so is the conclusion. It is a nice irony that when logicians try to prove the validity of *reductio ad absurdum* this way (as E.J. Lemmon does it in *Beginning Logic* while proving the completeness theorem for the propositional calculus), they use *reductio ad absurdum*.[11] Is there anything wrong with such a way of proceeding? It seems so. The problem is that it is hard to think of any other way of showing its validity, because of some kind of a meta-axiomatic quality it has. It turns out that we tacitly accept it and we think

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[11] E.J. Lemmon, *Beginning Logic*, Wokingham 1987, p. 80.

according to the way it works. This shows that RAA is one of the most primitive patterns of reasoning, so deeply rooted in our minds that, like other primitive entities, it does not require justification. We have been using it for millennia and will be doing so, simply because it stands to reason and we profit from it.

Not only the laws of logic or rules of inference deserve a closer look. Also, the metalogical properties of basic notions are worth comparing with natural ways of thinking. One of them is the monotonicity of classical consequence operation. It states that the larger the set of premises is, the larger the set of conclusions. In A. Tarski's exact formulation, where the set of all conclusions derived from X is denoted by Cn (X) and $\subseteq$ denotes set inclusion, monotonicity can be expressed as

If $X \subseteq Y$, then Cn $(X) \subseteq$ Cn (Y).[12]

This is certainly one of the most important features of many deductive systems. Once we prove something on the ground of a certain set of axioms, we shall also have the proof of that fact on the ground of an extended set of axioms. But in real life situations this property ceases to hold. From the premises stating that a public company announces terrific financial report and that it merges with the leader of the market we draw the conclusion that it is wise to buy its shares. But the additional premise, e. g. the information that the president of the company has been arrested on charges alleging creative accounting might make us change our mind and we might not be willing to stick to the earlier conclusion any longer.

The problem here is that the world of information we live in substantially differs from the world of formally derived facts. While the truths of the latter one are established once and for all, the ever-changing former one makes us less determined to uphold the conclusions we have arrived at earlier on. That is why non-monotonic logics have been introduced.

For similar reasons common sense prevents us from concluding the conjunction of two propositions that refer to the situations separated by time and thus conflicting with each other when one of them is out-of-date. Two pieces of information, equally credible: A = Tickets to the museum cost \$4, read in a tourist guide, and B = There is no entrance fee to the museum, found on the Internet, might be confusing. According to the classical &-introduction rule we should be able to conclude A&B, as classical logic does not distinguish the notions of past and present. Obviously, we do not do that, even though A, B $\vdash$ A&B is a truth-preserving rule. The way we reason in such cases is not 'classical' but 'temporal': we tacitly use the operators 'it was the case that', 'it is now the case that' or 'it will be the case that'. The concepts of past, present and future (occurring in temporal logic, which is actually used here) are substantial in such situations and prevent us from deriving absurd conclusions, e. g. that somebody lives and does not live at the same time (A & ¬A). In point of fact, the classical &-introduction rule in everyday reasoning is replaced by its more subtle tense-indexed version.

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[12] A. Tarski, 'Fundamentale Begriffe der Methodologie der deduktiven Wissenschaften I,' *Monatshefte für Mathematik und Physik* 37 (1930), p. 361–404.

One of the many virtues of formal logic is that it very often gives precise an-
swers to the questions that are intuitively hard to tackle. When we ask a person
untrained in logic to decide whether a sentence that contains empty terms (i.e.
terms that have no denotations) is true or false, he or she is usually puzzled. Cat-
egorical propositions like

> All unicorns are shod
> Some unicorns are shod

always pose problems for beginning students. The propositions are true and
false respectively, and their logical values are the consequences of the definitions of
basic propositional connectives. It is interesting that the latter proposition seems
less problematic while the value of the former is often hard to accept even for those
who are convinced by the formal explanation. Perhaps human minds by their very
nature are so informal that even when they enter the realms of fantasy, which are
hard to grasp by common sense, they still prefer to rely on their intuition rather
than the truths that are necessary by virtue of the logical form alone. That is to
say, intensional logic is used in such situations rather than the extensional logic of
objects that are determined by their members.

If we assume that students are statistically representative of the population,
some hints concerning natural logic can be obtained from the observation of the
beginning students attending the lectures in logic. Although some of the invalid
patterns of argument are properly recognised by them as invalid, others invariably
seem valid to them. The reversal of the conditional, $A \rightarrow B \vdash B \rightarrow A$, is usually
diagnosed correctly as invalid by a significant number of students (it is astonish-
ing that most of them give the same counterexample: squares are rectangles, but
rectangles are not necessarily squares). The situation changes when we consider
a very similar pattern of argument, namely $A \rightarrow B \vdash \neg A \rightarrow \neg B$ (it can be refuted
by the same counterexample). An observation that can be made here is that people
do not think in terms of the patterns of argument; instead, they think in terms
of particular cases, especially when those cases are close to their own experience.
Each student is eager to agree that If I study hard, I will pass my exams
therefore If I don't study hard, I will not pass my exams (or If the
weather is good, we shall go on a trip therefore If the weather isn't
good, we shall not go on a trip). He or she reasoned in these ways many
times and the results did not do much harm to them. Sometimes those richer in
academic experience have to verify their logic and are forced to weaken the conclu-
sion: If I don't study hard, it is very likely that I will not pass my
exams. The argument with this conclusion may be regarded as rational, however,
the propositional pattern of reasoning under which it falls still remains invalid.

As I mentioned earlier, each non-classical logic has its specific motivation, which
stems from a particular problem that a reasoning person may confront; in some cases
it is a contradiction, in other cases a time gap between the premises or the lack of con-
nection between the premises and the conclusion. Thinking informally or intuitively,
however, people use many tricks that are hard or impossible to formalise. While en-
countering two contradictory statements, people tend to ignore, sometimes wrongly,
the one that is less credible (formal logic in general does not evaluate the credibility

of statements), their reasoning is always well-anchored in time, and they never draw conclusions that have nothing to do with the statements taken as premises.

Most of the well-known attempts to approximate natural logic by a non-classical logic seem unsatisfactory to say the least. The reason is that formally we can tackle only one problem at a time: once we design a logic coping with one difficulty, we often find it impossible to cope with another, since by modifying our system appropriately we almost always obtain undesired by-products that substantially limit the range of the system. But non-classical logics have an immense cognitive value. The very work on them makes us better understand all the clever tricks of the trade of intuitive thinking, especially the thinking that has proved to be effective and beneficial to mankind. They serve a major role in bringing us nearer to natural logic, but not if considered separately. Only as a whole, can the variety of non-classical approaches bring us nearer to the answer.

3. The examples that I have presented here show that the laws, rules or properties of formal logic in some cases diverge from the natural ways of reasoning, and in other cases they coincide with them. Although I have mentioned only two instances of coincidence (*modus ponens* and *reductio ad absurdum*), it has to be stressed that such situations are decidedly more frequent. Formal solutions, in the vast majority of cases, precisely reflect the work of highly rational minds. One of the unavoidable problems is that logicians tend to formulate the laws of thought in the most general way possible for the purposes of deductive systems. Classical logic, for this instance, is a beautiful theory that serves other beautiful and exceptionally useful theories, namely those of mathematics. But in non-mathematical considerations some of those very general rules and laws lose their significance and become meaningless (excluded middle) or they are disregarded as being unintuitive (conditional introduction) or misleading (monotonicity). In order to make logic work more mind-like, logicians have to introduce non-classical amendments (intuitionistic, relevant, non-monotonic, etc.). Possibilities in that matter are practically infinite. This is why the work on formal logic is and will be the work in constant progress.

But will that progress ever get us substantially closer to the real picture of the natural ways of thinking? How can the variety of non-classical approaches considered as a whole have any practical value for us? These questions cannot be answered without a different attitude in our methodology or at least without changing an angle of looking at the problem. As I have mentioned earlier in this paper, the exploration of the nature of the mind's activity of the kind that we are interested in would require both logical and psychological expertise. Such a research may provide us with new formal systems, on the one hand, and illuminating knowledge of how our minds work, on the other. The problem definitely requires a specifically oriented interdisciplinary approach and it seems extremely interesting to explore the matter of these two aspects of reasoning simultaneously in a more systematic way. On their part, logicians have an important role to play: to make further discoveries and inventions, keeping our minds in their minds.[13] Psy-

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[13] An illuminating insight into these matters can be found in K. Devlin, *Goodbye Descartes — The End of Logic and the Search For a New Cosmology of the Mind*, New York 1997.

chologists should be able to tell discoveries from inventions. There should be also other benefits, both to logicians and psychologists. G.B. Keene, analysing differences between classical and non-classical logic, points out that it is a disagreement about the epistemic concept 'has deductive warrant for concluding' that is at issue here.[14] He also adds that: "[...] a psychological investigation into the resistance to, or acceptance of, inferences involving relevance fallacies might, itself, be of considerable interest to both disciplines."[15]

It goes without saying that the quality of informal or intuitive thinking depends on one's intelligence.[16] Intelligent thinking, however, does not mean logical thinking. We have to think logically to be intelligent but logical thinking is not sufficient. As D. Miller points out, in order to be effective in problem solving, we have to think creatively or speculatively, and informed guessing (our guesses should be informed by earlier guesses) plays a substantial role in this process.[17] According to E. de Bono, due to its verticality (premises-conclusion scheme), logical thinking lacks the creative factor — a necessary part of so-called lateral thinking, i.e. thinking that makes us able to find various solutions to all sorts of problems through an indirect and creative approach that is not immediately obvious.[18] In his opinion it is impossible to formalise lateral thinking, since in this case logic follows the mind, while in vertical thinking the mind follows logic. Therefore, the question about the natural logic of human mind in all probability will not be answered in the language of formal logic, since we are not able to express in that language the entirety of the phenomenon of human intelligence. Hence, the first step on our way to modelling the natural logic in all its complexity (understood not just as a set of truth-preserving rules) seems to be finding brand new extensions of the formal language. Again, possibilities are infinite and many steps in that direction have already been taken (like those in the research on artificial intelligence, informal logic and argumentation theory, e.g. see M. Janier, M. Aakhus, K. Budzynska, C.Reed[19]). But what I am interested in is an enterprise on a smaller scale: to get to know the relation between the natural and formal logic limited to the set of rules, laws, and properties, and possibly to use this knowledge in further formal investigations.

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[15] Ibidem, p. 62.

[16] The psychological analysis which exposes the traps of intuitive thinking can be found in D. Kahneman's *Thinking, Fast and Slow*, New York 2011.

[17] D. Miller, 'Do We Reason When We Think We Reason, or Do We Think?,' *Learning For Democracy* 1 [3] (2005), p. 57–71.

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